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Bifurcation analysis in the flight dynamics design process? A view from the aircraft industry

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Since 1977, various studies have been conducted to analyse the dynamics of aircraft by using continuation schemes and bifurcation analysis. The technique has not, however, been adopted as an engineering tool in the industry. After reviewing previous work, we apply bifurcation analysis to an industrial-scale aircraft model. The technique is shown to locate and identify key changes in the dynamic behaviour of the aircraft as its flight controls are varied. The issues facing the aircraft industry in implementing this technique are raised and a set of software improvements suggested. With these improvements the technique could be made into an engineering tool with important benefits to industry.

Keywords: industry application; bifurcation analysis; software requirements; software improvements; AUTO94

1. Introduction

As described in the introduction to this issue, simulation remains the key (and usually the only) nonlinear analysis tool in the flight dynamics industry. In 1977 bifurcation analysis was applied to flight dynamics for the first time (Mehra *et al*. 1977, 1978, 1979). Since then, numerous papers have demonstrated the technique's ability to present a picture of an aircraft's dynamic behaviour, different from but complementary to, simulation (Avanzini & de Matteis 1996; Barth & Planeaux 1988; Baumann 1989; Carroll & Mehra 1982; Davison 1992; Gao *et al*. 1988, 1990; Goman & Zagaynov 1984; Goman & Khramtovsky 1995; Guicheteau 1990, 1993*a*, *b*; Hawkins 1985; Jahnke & Culick 1988, 1994; Jahnke 1990; Liebst & Nolan 1993; Littleboy & Smith 1996; Lowenberg 1991*a*, *b*, 1994*a*, *b*, 1996; Macmillen 1995, 1996; Mehra *et al*. 1979; Nolan 1992; Planeaux *et al*. 1990). However, for new techniques such as bifurcation analysis to be accepted by, and of benefit to, the industry, several criteria must be met:

1. industry must know of the technique's existence;

2. industry must understand the technique;

3. industry must be convinced that the technique can either improve performance (in the widest sense of the word), or decrease process time (and hence cost);

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Figure 1. The Harrier II. (Photograph courtesy of Richard Clairoux.)

4. industry must be provided with a tool which requires a minimal amount of effort to use, and setting up and development costs must not outweigh the perceived benefits.

As a follow-up to work that addresses the first pair of issues (Macmillen 1995), this current work aims to address the second pair of issues. To this end, the bifurcation-analysis technique has been applied to an industrial-scale aircraft model, in order to identify the key benefits, limitations and developments required.

2. The Harrier model

The British Aerospace/McDonnell Douglas Harrier II has been chosen for this study. It is known to exhibit a wide range of dynamic behaviour and a complete 'industrial-scale' nonlinear simulation model is available.

While only an eight-state model has been used (no control system has been included which increases the number of states), the model is generally more complex than many aircraft models studied in the literature.

1. The full rigid-body equations of motion are used without simplification. Cross-products of inertia and variation of centre-of-gravity (CG) position are catered for within the equations of motion.

2. The model is not aerodynamically symmetrical, i.e. at zero sideslip there exist lateral-directional forces and moments. This immediately precludes symmetry-breaking bifurcations (such as the pitchfork) that have been identified in other applications to aircraft models (Guicheteau 1993*a*).

3. In the language of the flight dynamicist, the aircraft has an unstable spiral mode. This means, in the language of the nonlinear dynamicist, that there exists a stable mode characterized by a spiral-type motion. Hence, unlike nearly all studies to date, the aircraft's main equilibrium branch with lateral/directional controls (aileron/rudder) neutral, corresponds to a steady spiral dive with constant sideslip, roll rate and yaw rate, i.e. no purely longitudinal steady-state motion exists. This may be compared to the results by Jahnke (1990) for example.

4. The aerodynamic model used consists of 26 multi-dimensional look-up tables containing over 7000 data points.

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5. In the current study the stability augmentation system has not been modelled, but the auto-flap system has been included.

All analysis presented here has been performed at a single thrust, CG and inertial condition. In practice, CG, mass or inertia may be varied or be used as a continuation parameter.

3. Interpolation

The complete aerodynamic model for the Harrier consists of over half a million data points contained in 56–130 tables (depending on configuration) of two, three and four dimensions. This is representative of the complexity of aircraft models used in the aircraft industry. However, the AUTO94 code used for continuation is unable to accommodate linear interpolation of these tables, hence a tensioned spline algorithm has been written to interpolate one- and two-dimensional tables. The model has therefore been simplified by limiting tables to two independent variables, i.e. by neglecting second-order effects. While this simplification has detracted from the aim of making the task as realistic as possible (from an industrial viewpoint), lessons have still been learned, and are described below.

A rational spline is a piecewise polynomial with rational coefficients. Cubic splines are the most common interpolation spline used by engineers; they ensure smoothness (continuity of the second derivative) across knot points. However, they often exhibit excessive curvature between knot points, i.e. they appear too ‘wiggly’. A tensioned spline is an adaptation of a cubic spline, which allows the user to apply a tension parameter to iron-out any ‘wiggles’. Tensioned splines take longer to construct (to determine the spline coefficients), but these coefficients may be stored ready for interpolation. A tensioned spline is almost as fast to interpolate as a normal cubic spline. The rational spline algorithm developed by Spath (1974) has been used. During a continuation sweep, the independent variable will generally be in the same interval between knot points as in the previous interpolation pass. Therefore the interpolation scheme checks that the independent variable lies in the same interval as previously, before starting a search for the correct interval.

Interpolation of aerodynamic data is an important issue in industrial-scale applications. A typical model, for example, may contain 100 aerodynamic tables, a typical continuation range may be 10° control-surface deflection with a step size of 0.01; for each step, four Newton iterations may be required. This results in 400 000 interpolations to trace out a typical equilibrium branch. For a periodic orbit this may increase by two orders of magnitude, giving 40 million interpolations. By comparison, a simulation run of 100 s with a 0.01 s step size and fourth-order Runge–Kutta integration would give four million interpolations. From this, it may be deduced that the interpolation scheme used within a model must be fast.

4. AUTO94

The AUTO94 continuation and bifurcation-analysis software (Doedel & Kernevez 1994) has been chosen for its availability, zero cost, compatibility with existing hardware and software, and the comprehensiveness of its algorithms. AUTO94 is a FORTRAN code first developed in the early 1980s and since updated; however,

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no program history is available. For this study the features of the code that are of interest are

1. continuation of stable and unstable equilibria branches and identification of saddle-node, transcritical, pitchfork and Hopf bifurcations;
2. continuation of stable and unstable periodic solutions and identification of cyclic folds, cyclic pitchforks, period-doubling and secondary Hopf bifurcations; and
3. continuation of fold and Hopf bifurcation points in two parameters.

Certain software changes and additions had to be made to the AUTO94 code in order to analyse the Harrier model. First, with models containing more than five dimensions, the AUTO code must be edited to account for additional dimensions. Secondly, the user must describe the aircraft via a subroutine 'FUNC'. This subroutine has been written to use the full body-axis equations of motion, solved by Gaussian elimination. The subroutine also contains calls to the aerodynamic interpolation routines and converts from body-axis variables, u, v and w, to the stability axis state variables V, α and β. A standard Newton scheme has been used to find and obtain starting solutions. It should be noted that AUTO94 requires an extremely accurate starting condition in order for the continuation scheme to proceed. Double precision is mandatory; this is an issue in industry, where existing simulation models and code may have been written with single precision. A search routine has also been written which produces a grid of starting points from which the Newton scheme tries to find a solution. This technique is crude, time consuming to run, and does not ensure that all possible starting solutions are found. The 'SSNE' technique described by Goman & Zagaynov (1984) for obtaining starting solutions may provide a more efficient and 'safe' technique. Both approaches should be compared back-to-back for an accurate comparison.

AUTO94 is a generic program, and the user must set 36 parameters to define the problem and the operation of AUTO94. The following parameters are of particular importance.

DSMAX The maximum allowable pseudo-arclength step size requires setting to ensure convergence with acceptable speed. Values ranged from 0.5 to 0.01. An adaptive mesh and step size has been used for all runs.

DSMIN The minimum allowable step size has been kept at 1×10^{-6} for all runs.

ITMX This sets the number of Newton iterations for accurate location of bifurcations. It was set high at 50.

ITNW The number of Newton iterations before changing arclength step size has generally been set to seven.

NTST For periodic orbits, NTST sets the number of mesh points for the collocation scheme used. NTST must be as small as possible for speed, but if set too low it results in the continuation scheme reversing direction at bifurcation points. A value of 50 has usually been used, a value of 20 usually being too low.

ISP This must be set to two in order for AUTO to identify period-doubling and secondary Hopf bifurcations.

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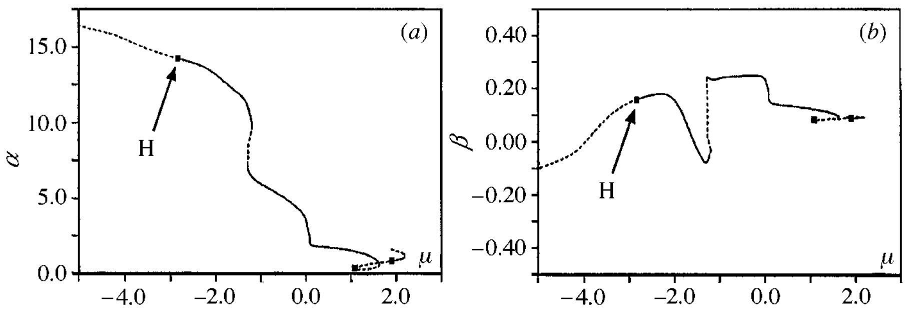

Figure 2. The AOA (α), and angle of sideslip (β) versus stabilitor: main equilibrium branch.

5. Bifurcation analysis of the Harrier model

All results presented are with the horizontal stabilitor (μ), the longitudinal (pitch) aircraft control, as the continuation (control) parameter. All other aircraft controls have been held constant (ailerons, rudder, thrust, etc.). On the bifurcation diagrams, solid lines denote stable equilibria, dashed lines unstable equilibria. Solid circles denote stable periodic solutions, open circles unstable periodic solutions. Supercritical Hopf bifurcations are identified by solid squares.

(*a*) *Main equilibrium branch*

The first bifurcation diagram, figure 2, is from the most complete model analysed. It shows the main equilibrium branch which corresponds to the normal flight regime of the aircraft. Here two of the eight state variables are shown. As the pilot moves the stabilitor (μ) to negative deflections, the angle of attack (AOA) (α) increases. At $\alpha = 8$–$10°$ is an unstable region bounded by fold bifurcations. This corresponds to a small aerodynamic pitch instability in the basic aircraft model.

Angle of sideslip (β) is small, as expected in normal flight. It is non-zero, however, due to small aerodynamic asymmetries included in the model.

The Harrier aircraft has a spiral mode that is stable in the dynamical system sense (but unstable in the terminology of flight dynamicists): i.e. left to its own devices the aircraft will enter a steady downward spiral. Hence, the three rotational rates, roll (p), pitch (q) and yaw (r), are all non-zero but under $10° \text{ s}^{-1}$ in the stable region of figure 2. The main equilibrium branch becomes unstable at low α by means of a fold, and at moderate α by a supercritical Hopf bifurcation (H). Further analysis concentrates on the dynamics at the higher AOA, more commonly reached during aggressive manoeuvring.

(*b*) *Periodic solutions*

The following of the periodic orbit arising from the Hopf bifurcation at moderate α has initially been performed on a simplified model. This simplified model gives the same qualitative dynamics as the more complex model in the vicinity of the Hopf bifurcation. The supercritical Hopf (H) yields a stable limit cycle which almost immediately undergoes a couple of cyclic folds to give a stable periodic solution (X). These cyclic folds result in an effective jump to an oscillatory state as the pilot increases his AOA by decreasing μ. The maximum values of two state variables in this stable periodic solution are shown in figure 3.

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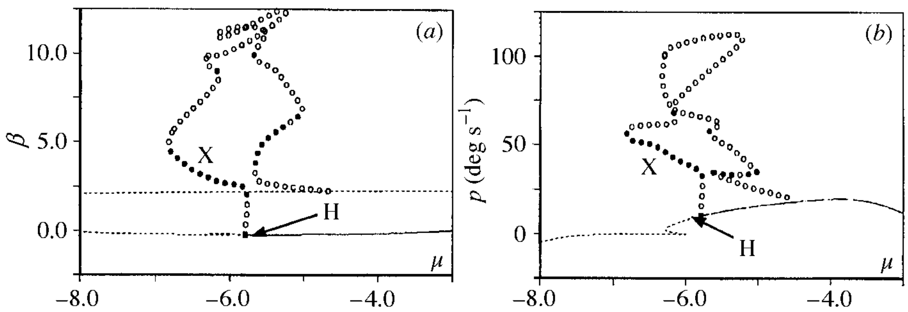

Figure 3. Peak angle of sideslip (β) and roll rate (p) versus stabilitor.

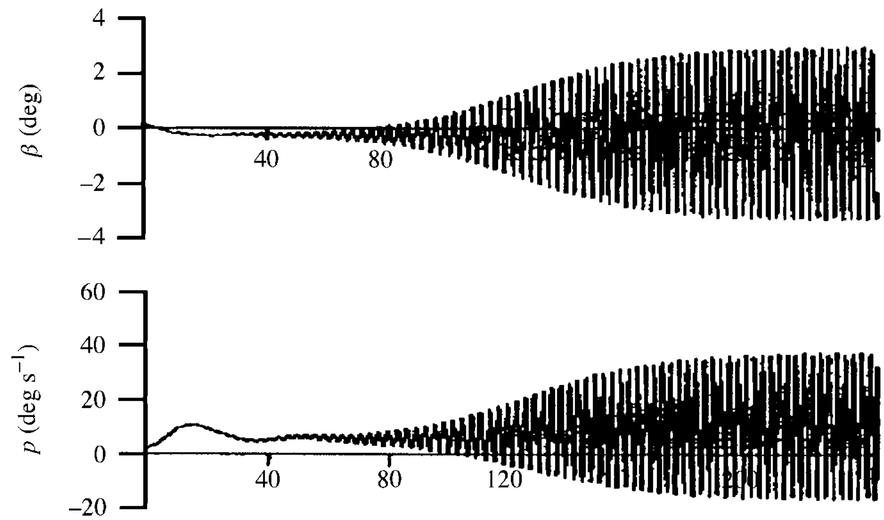

Figure 4. Angle of sideslip (β) and roll rate (p) versus time in seconds, following the Hopf bifurcation.

In the simplified model, the jump occurs at about $\alpha = 17°$. The resulting oscillation is a lateral-directional mode characterized by high roll rates of 30–50° s^{-1}. The period is 2.7 s. Such a motion is known as 'wing rock'. A time-history of the wing-rock development for the same two state variables is shown in figure 4. Harriers can exhibit such 'wing-rock' behaviour in wind-up turns in which the pilot progressively increases his AOA. While the α from flight tests at which onset of wing-rock occurs agree well with the above analysis, such wing rock is not usually sustained in flight, and comparison of steady-state characteristics is more difficult. However, wing rock develops when the complex eigenvalues corresponding to a 'Dutch-roll' mode become unstable. The frequency of Dutch roll on Harriers is typically 2–3 s, which agrees well with the time-period of the wing rock. Indeed, it is through bifurcation analysis that the link between Dutch roll and wing rock was conclusively proven (Liebst & Nolan 1993). This link between Dutch roll and wing rock is examined in the paper by Liebst (this issue).

The above example demonstrates how continuation and bifurcation analysis may be used to find important changes in an aircraft's dynamic behaviour, and focus

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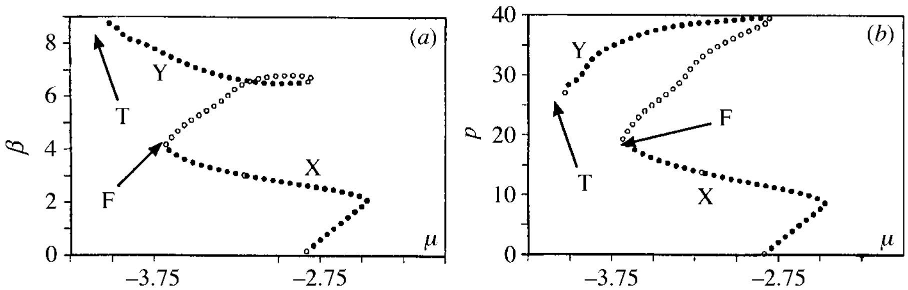

Figure 5. Peak angle of sideslip (β) and roll rate (p) for multiple periodic solutions.

simulation effort. In return, the simulation demonstrates how slowly (or otherwise) the dynamic behaviour changes during the transient motion.

(*c*) *Toroidal solutions*

The above results are from a simplified aerodynamic model. The final model analysed contained all major aerodynamic terms and the auto-flap control system, which is usually engaged in flight. Following the resulting periodic orbit with this more-comprehensive model reveals more complex behaviour.

Again, a pair of cyclic folds yields a jump to stable wing rock (X), the magnitude of oscillation increasing as μ is decreased. At $\mu = -3.7°$ ($\alpha = 15.5°$) a cyclic fold (F) gives an unstable branch which eventually doubles back via another cyclic fold to give a new stable periodic branch (Y) but with much increased rotational rates, sideslip and velocity (figure 5). With the loss of a locally stable solution at $\mu = -3.7°$, the aircraft will jump to the new periodic solution (Y) as the control is moved past $-3.7°$. This jump is shown in a simulation time-history (figure 6), in which the control μ has been varied slowly.

In the AUTO94 software, it is not currently possible to record the minimum value of each state in a periodic orbit easily. However, for asymmetric motions, as is the case here, the minimum state value may have greater magnitude and importance than the maximum value.

If μ (and α) are increased further, the periodic solution becomes unstable, via a secondary Hopf bifurcation (T) to a torus. This behaviour contains two periods, one very close to the wing rock, 2.7 s, and one considerably slower, 19.2 s (figure 7). Such toroidal behaviour has been demonstrated before in aircraft models (Goman & Zagaynov 1984). Plots of phase-space projections illustrate the toroidal nature of the motion (figure 8).

6. Benefits

As with previous studies, the above results show that continuation and bifurcation analysis are able to highlight areas of interest for simulation analysis. They provide a more global picture than simulation, one which sets the context in which changes in dynamic behaviour may be explained and understood. Having used the technique, we would argue that a better 'feel' for the dynamics of the aircraft is obtained if bifurcation analysis is used in conjunction with simulation. Other studies have

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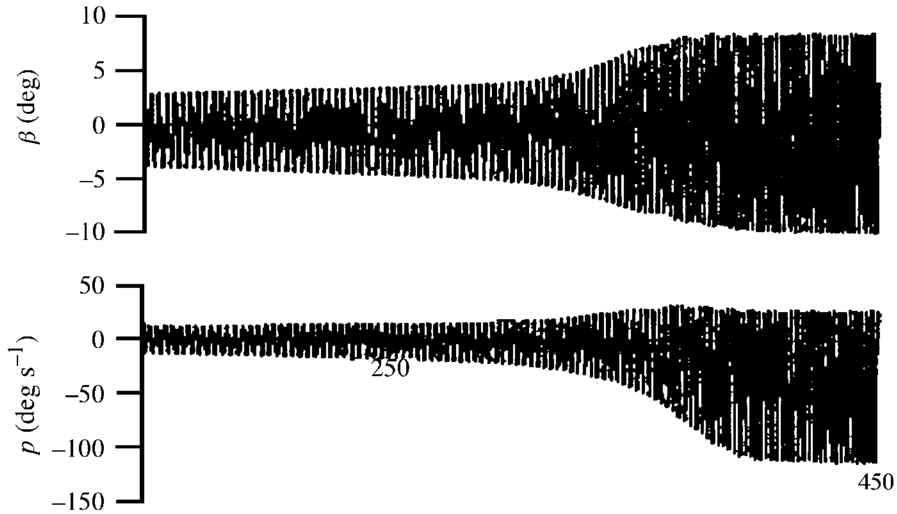

Figure 6. Angle of sideslip (β) and roll rate (p) versus time in seconds: jump between periodic solutions.

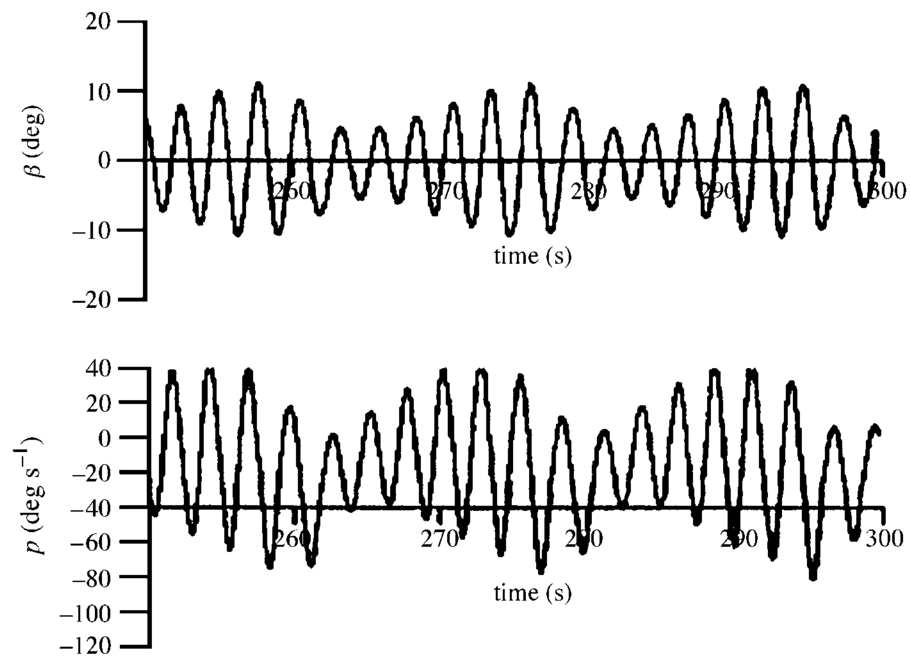

Figure 7. Toroidal motion time-history.

begun to demonstrate the technique's potential for control-law design. To what extent therefore would the technique benefit industry?

To clear an aircraft for flight test requires obtaining as close to a global and complete picture of the aircraft's dynamics as possible; and ensuring that this behaviour is acceptable. Combat aircraft fly in a large phase space. Flight condition is dependent upon altitude, Mach number, CG, inertia and external stores configuration. There are

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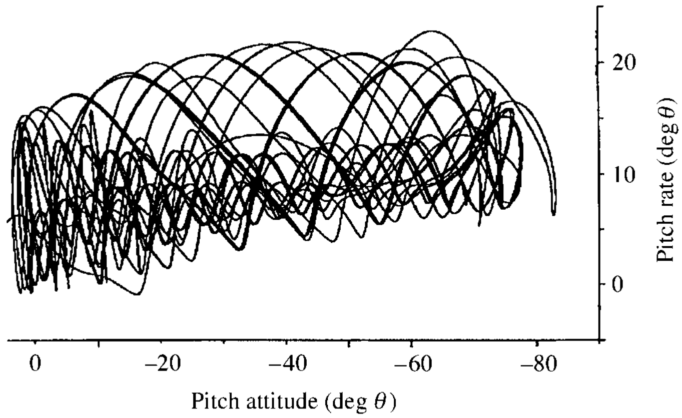

Figure 8. Toroidal motion phase projection

multiple control surfaces which may be moved in almost any combination, modern flight-control systems may operate in several different modes, and failure cases must also be considered. Flight clearance is currently performed through repeated nonlinear simulation. It may typically represent about 60% of the total cost of a project's aerodynamic content and over 40 human-years may be spent on flight clearance of a combat aircraft. Hence a reduction of say 30% in this process time would save in excess of 10 human-years per project. Could bifurcation analysis offer such a saving?

In the first place, the software would have to be developed so as to use the same aircraft model as the simulation tool, thereby eliminating additional set-up costs. One may then consider the ability of the technique to reduce the amount of simulation, and analysis, required. Continuation of equilibrium branches provides insight into where bifurcations occur and the qualitative dynamics to be expected. Such a knowledge could potentially result in savings of the order of 30% of simulation time. However, such a saving is dependent upon certain improvements to the software used.

Application may also be found in flight-control system design. Control-law design is almost exclusively performed by using linear design methods, whether they be classical or modern control techniques. Problems often arise when control laws are implemented within a nonlinear model. Nonlinear simulation is the principal tool for analysing nonlinear control systems. Bifurcation analysis may be applied to both preliminary control-law layout and to detailed/corrective design. First, these applications studies have focused on using bifurcation analysis in comparing the use of control effectors or feedback schemes when multiple options exist (Lowenberg 1994*b*, 1996; Planeaux *et al*. 1990). The aim is to compare the effect of different schemes on the stability of the aircraft, i.e. its bifurcational behaviour. Second, these techniques may be used to aid design of a control law to address a specific problem area, such as a wing-rock suppression system (Planeaux *et al*. 1990). The technique could alternatively be used to design/assess a spin-prevention or recovery system. In such cases, the efficacy of the potential scheme, and its impact on the global dynamics of the aircraft, may be checked.

The above is all very well, but current practice is much closer to the approach adopted by Liebst (this issue). Liebst uses bifurcation analysis as a 'specialist' tool

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for a specific purpose. His aim is to determine a simple parameter for the prediction of wing rock for any aircraft, a parameter which may be estimated easily by any flight dynamicist without having to resort to a complex analysis package such as AUTO. As he points out, similar parameters exist for the prediction of departures and spin susceptibility and such techniques are useful. Therefore, it seems, the added benefit of seeing the 'global picture' from bifurcation analysis is only worthwhile if it may be achieved relatively easily. As Liebst says, his aim is to predict wing rock 'without using complicated software and costly computer time'. If bifurcation analysis-type techniques are to be used in industry, the software must not be complicated or too costly to run, and it must interface with existing aircraft models already created for simulation.

7. Developments

The following amounts to a specification for an 'ideal' software tool which would be suited to use in industry.

(*a*) *Software control*

If the tool is to be used for flight clearance of aircraft, it must both be validated and configuration-controlled.

(*b*) *Model definition*

The tool must interface directly with existing aircraft models used for simulation analysis. Clearly, this interface will vary between companies, and industrial participation may be required to develop such interfaces. What must be avoided is having to create a distinct (but essentially the same) aircraft model for the bifurcation analysis.

(*c*) *Continuation scheme*

The pseudo-arclength continuation method used in AUTO94 is unsuitable. A robust continuation method is required that converges for non-smooth systems, i.e. for models with linearly interpolated aerodynamics and discontinuities such as rate and amplitude limits. Such schemes are being incorporated into other software packages (KRIT), and have been used successfully on aircraft models (Avanzini & de Matteis 1996; Goman & Khramtsovsky 1995). Ultimately, the software should be compatible with the same aircraft model as is used for nonlinear simulation.

(*d*) *Speed*

For this study, AUTO94 was run on an SGi Iris. This gave acceptable performance for equilibria paths but was too slow for periodic solutions. A typical periodic branch took 2–4 h to compute. Considering that a complete aircraft model may be two to three times as large, run times would be 'overnight' in length. By far the greatest computational time (67%) was spent in interpolation of aerodynamic tables. A further 8% was spent on pivoting and Gaussian elimination. Speed may be increased by being able to use linear interpolation and optimizing the search/interpolation routines. The interpolation task is also highly suited to parallel processing, and this could achieve an improvement in speed of two orders of magnitude. Such an increase in speed is needed as the software is best used interactively.

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(*e*) *Comprehensiveness*

AUTO94 is good in terms of its ability to identify all bifurcations of interest; its restart, branch-switching and bifurcation-following capabilities are all required. Some important improvements would include recording maxima and minima of periodic orbits in all state variables in a single continuation sweep; making the ability to increase the dimensions of the model straightforward; importing the ability to control the output of information; and improving the versatility of the graphical output.

(*f*) *Ease of use*

The graphical user interface (GUI) of AUTO94 is of benefit. However, the program, because of its generality, is difficult to set up and use on a new application without prior experience. Software developed for industry would require parameter settings or guidelines specific to the application.

(*g*) *Basins of attraction*

Knowing basin boundaries would complete the picture given by bifurcation analysis. There are two issues, however.

1. How easily could knowledge of basin boundaries be used to aid simulation/flight clearance, or control law design?
2. How are the boundaries to be calculated?

The first issue is the dominant one: for an unaugmented aircraft it would be possible to look at projections in all eight state variables and rapidly determine in which states the boundaries lie closest to the equilibrium. This would give a 'manoeuvre' boundary within which the aircraft would not jump to a different steady-state solution. For an aircraft with control augmentation, the number of states increases rapidly and the picture may become too complex. Some automated technique for identifying and displaying the most important phase projections could be required. Basins of attraction may be determined via 'carpet-bombing' techniques (Goman & Khramtsovsky 1995), but this is computationally inefficient. Further research is required to look at ways in which boundaries may be rapidly estimated to sufficient accuracy.

(*h*) *Experience*

Two decades of experience in this application have been gained, but this experience needs to develop in the following two ways.

1. More experience is required on the application to large aircraft models with complex control systems within an industrial environment. A control system increases dimensions and complexity and gives rise to additional problems: multiple solutions for the same control parameters; speed of computation; and finding all solution branches.
2. Engineers in industry often face a terminology gap in the application of modern nonlinear dynamics theory applied to aircraft dynamics. The technique has to be both explained and demonstrated to be beneficial, for it to be adopted by industry.

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8. Conclusions

We have found that bifurcation analysis provides an insight into an aircraft's dynamic characteristics which is different from, but complementary to, nonlinear simulation. The technique has clear abilities to trace out all the trim states of an aircraft, and locate and identify qualitative changes in its dynamics. These abilities may be used to reduce the amount of simulation time required to clear an aircraft for flight. They may also be useful aids to control law design. However, suitable software is required to implement these techniques efficiently in an industrial environment. A software 'specification' has been offered to meet the needs of industry. Due to the large cost of both flight-clearance and control-law design, there is an incentive to develop the techniques and software in order to provide such a tool for industry. After two decades, it is time for the industry to have an input into the future development of the technique and its software, to assess the technique, and hopefully benefit from it.

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